REGULAR REFLECTION OF WEAK SHOCK WAVES FROM A RIGID WALL

(O REGULIARNOM OTRAZHENII SLABYKH UDARNYKH VOLN OT ZHESTKOI STENKI)

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Problems of regular reflection of weak shock waves from a rigid wall cannot be investigated with the aid of the acoustic theory alone, since it is necessary to take account of the dependence of the basic parameters of the flow on the overpressure. The general principles of the theory of short waves, considering this dependence in the first approximation, were developed by Ryzhov and Khristianovich in [1].

In the same paper the problem of regular reflection in its nonlinear formulation was investigated for the first time. Exact solutions of a reflected system of short waves were used for an approximate solution of the problem. In this connection, the arbitrariness of the constants contained in the solutions was used to satisfy the boundary conditions approximately.

In the present paper, the method of expansion of the unknown functions in series in a small parameter is used to solve the equations of short waves. The magnitude of the overpressure is taken as the small parameter. The form of the boundary condition which the solutions of the equations of short waves must satisfy at the shock front are presented. In the investigation of the problem of regular reflection, particular solutions of a system of short waves are sought in the form of a direct expression of the flow velocities as functions of the coordinates. This permits us to satisfy fairly accurately the condition of conservation of tangential velocity across the front of the reflected wave and all the other boundary conditions. The simple analytical form of the solution permits us to integrate the differential equation of the reflected shock wave, to find the coordinates of the front in closed form, and also to trace the continuous variation of the entire pattern of the reflection and of the velocity field as the basic data are varied in the range of regular reflection. Examples are given of the analysis of the flow in the case of a noncritical and a critical value of the initial data.

1. We shall present a derivation of the equations of short waves which is based on the expansion of the unknown functions in series in the small parameter $P = p/(nP_0)$, where P is the overpressure, P_0 is the initial pressure, and n is the constant ratio of specific heats (for air $P_0 = 1$ atm, $n = 1.4$).

The equations of motion of a compressible gas for plane flows have the following form in cylindrical coordinates r, ϑ :

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial r} + \frac{v}{r}\frac{\partial u}{\partial \vartheta} - \frac{v^2}{r} + \frac{1}{\rho}\frac{\partial p}{\partial r} = 0$$
$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial r} + \frac{v}{r}\frac{\partial v}{\partial \vartheta} + \frac{uv}{r} + \frac{1}{\rho r}\frac{\partial p}{\partial \vartheta} = 0 \qquad (1.1)$$
$$\frac{\partial \rho}{\partial t} + u\frac{\partial \rho}{\partial r} + \frac{v}{r}\frac{\partial \rho}{\partial \vartheta} + \rho\left(\frac{\partial u}{\partial r} + \frac{1}{r}\frac{\partial v}{\partial \vartheta} + \frac{u}{r}\right) = 0$$

Here u and v are the projections of the velocity vector q in the directions of the radius vector and the perpendicular thereto, p is the overpressure, ρ is the density, t is time.

For weak shock waves, the process of compression may be considered to be very nearly adiabatic, which permits us to write the pressure-density relation for air in the form

$$p = P_0\,[(\rho / \rho_0)^n - 1] \qquad (1.2)$$

where ρ_0 is the initial density.

Proceeding from the last relation, we shall now find the leading terms of the expressions for the basic parameters of the flow in series in the small values of P .

From (1.2), we have for the density

$$\rho / \rho_0 = 1 + P - {}^1/_2\,(n-1)\,P^2 \qquad (1.3)$$

We have the shock conditions (dynamic compatibility) (1.4)

$$\rho\,(N - q_n) = \rho_1\,(N - q_{1n}),\quad p - p_1 = \rho_1\,(N - q_n)\,(q_n - q_{1n}),\quad q_\tau = q_{1\tau}$$

for the velocity of propagation of the wave front N in a medium with overpressure P_1 and particle velocity q_1, and also the normal and tangential components q_n and q_τ of the particle velocity q behind the front of the shock wave.

Applying (1.3) to these, we have, following [2],

$$N = a_0[1 + {}^1/_4\,(n+1)\,P + {}^1/_4\,(n-3)\,P_1] + q_{1n}$$
$$q_n = a_0\,(P - P_1) + q_{1n} \qquad (1.5)$$

If we consider that the overpressure P_1 and velocity q_1 correspond to the state behind the fornt of a wave N_1 which propagates into undisturbed gas having zero overpressure, then $q_{1n} = a_0 P_1$, and (1.5) assumes the form

$$N = a_0\,[1 + {}^1/_4\,(n+1)\,(P + P_1)], \qquad q_n = a_0 P \qquad (1.6)$$

Let $r = r\,(\vartheta, t)$ be the equation of the front of the shock wave, ψ the angle between the normal to the shock front and the direction of the radius vector, θ the angle between the direction of the radius vector and that of the particle velocity. We then have the following equations for the components u and v at the wave front:

$$u = q_n \cos\psi + q_\tau \sin\psi, \qquad v = q_n \sin\psi - q_\tau \cos\psi \qquad (1.7)$$
$$q_n = q\cos(\psi - \theta),\quad q_\tau = q\sin(\psi - \theta),\quad \tan\theta = v / u,\quad \tan\psi = r^{-1}\,\partial r / \partial\vartheta$$

On the line of discontinuity, for small values of the angle ψ and θ, we shall have

$$u = q\,(1 - {}^1/_2\psi^2), \qquad v = -u\psi, \qquad \partial r / \partial t = N\,(1 + {}^1/_2\psi^2) \tag{1.8}$$

The last one of these is obtained by equating two expressions for the velocity of propagation of the shock front in the direction of the radius vector, $\partial r / \partial t = N \sec\psi$.

We now introduce the dimensionless functions M and V in accordance with [1] and the independent variables Δ, Y, τ by means of the relations

$$M = \frac{u}{a_0}, \qquad V = \frac{v}{a_0}, \qquad \Delta = \frac{r}{a_0 t} - 1, \qquad Y = \frac{\vartheta}{\theta_0 \sqrt{{}^1/_2\,(n+1)}}, \qquad \tau = \ln t \tag{1.9}$$

where θ_0 is some characteristic value of the angle. The third equation of (1.8) will then assume the following form upon substitution of the definitions of (1.9) and omission of terms known to be small:

$$\Delta + \frac{\partial \Delta}{\partial \tau} = \frac{n+1}{4}\,(P + P_1) + \frac{1}{(n+1)\,\theta_0^2}\left(\frac{\partial \Delta}{\partial Y}\right)^2 \tag{1.10}$$

Considering $\tau \sim Y \sim 1$, it follows from the last equation and the remaining ones of (1.8) that

$$\Delta \sim P, \qquad \theta_0 \sim P^{1/2}, \qquad M \sim P, \qquad V \sim P^{3/2} \tag{1.11}$$

The relations which we have obtained determine the orders of magnitude of the parameters of the flow at the shock front. We shall consider that these orders of magnitude are also retained in some region adjoining the shock wave front.

After small quantities of higher order are dropped, the equations of motion (1.1) in terms of the variables of (1.9) assume the form

$$\begin{gathered}\frac{\partial M}{\partial \Delta} = \frac{\partial P}{\partial \Delta}, \qquad \frac{\partial V}{\partial \Delta} = \frac{1}{\theta_0 \sqrt{{}^1/_2\,(n+1)}}\,\frac{\partial P}{\partial Y} \\ \frac{\partial M}{\partial \tau} + \frac{\partial P}{\partial \tau} + (M - \Delta)\left(\frac{\partial M}{\partial \Delta} + \frac{\partial P}{\partial \Delta}\right) + (n-2)\,P\,\frac{\partial P}{\partial \Delta} + \\ + P\,\frac{\partial M}{\partial \Delta} + \frac{1}{\theta_0 \sqrt{{}^1/_2\,(n+1)}}\,\frac{\partial V}{\partial Y} + M = 0\end{gathered} \tag{1.12}$$

Integrating the first equation of the system (1.12), we have $M = P + F(Y)$, where $F(Y)$ is an arbitrary function. On the shock wave front we have $M = (1 - {}^1/_2\psi^2)\,P \approx P$, according to (1.8) and (1.6); therefore $F(Y) = 0$. Equations (1.12) then take the form

$$\begin{gathered}M = P, \qquad \frac{\partial V}{\partial \Delta} = \frac{1}{\theta_0 \sqrt{{}^1/_2\,(n+1)}}\,\frac{\partial M}{\partial Y} \\ \frac{\partial M}{\partial \tau} + \left(\frac{n+1}{2} M - \Delta\right)\frac{\partial M}{\partial \Delta} + \frac{1}{2\theta_0 \sqrt{{}^1/_2\,(n+1)}}\,\frac{\partial V}{\partial Y} + \frac{1}{2} M = 0\end{gathered} \tag{1.13}$$

which is basic for the investigation of short waves.

In order to investigate the system (1.13) taking account of the estimates

(1.11), it is convenient, following [1], to introduce

$$M = M_0\mu, \qquad V = M_0\sqrt{1/_2\,(n+1)\,M_0}\,\nu, \qquad \Delta = 1/_2\,(n+1)\,M_0\delta$$
$$\theta_0 = \sqrt{M_0} \tag{1.14}$$

where M_0 is a characteristic value of the number M. The system of equations for short waves then assumes the form

$$\frac{\partial\mu}{\partial\tau} + (\mu-\delta)\frac{\partial\mu}{\partial\delta} + \frac{1}{2}\frac{\partial\nu}{\partial Y} + \frac{1}{2}\mu = 0, \quad \frac{\partial\nu}{\partial\delta} - \frac{\partial\mu}{\partial Y} = 0, \quad M = \frac{p}{nP_0} \tag{1.15}$$

Here in the case of self-similar flows $\partial\mu/\partial\tau = 0$.

2. The differential equation which determines the position of the shock front is obtained from (1.10) by introducing the notation of (1.14)

$$\frac{\partial\delta}{\partial Y} = \pm\left\{2\left[\delta\left(1+\frac{1}{M_0}\frac{\partial M_0}{\partial\tau}\right)+\frac{\partial\delta}{\partial\tau}\right]-(\mu+\mu_1)\right\}^{1/2} \tag{2.1}$$

We introduce further, in accordance with [1], a moving coordinate system

$$x = a_0t\,([1 + 1/_2\,(n+1)\,X] \approx a_0t\,(1 + \Delta - 1/_2\vartheta^2)$$
$$y = a_0t\sqrt{1/_2\,(n+1)\,M_0}\,Y \approx a_0t\vartheta, \qquad \delta = X + 1/_2Y^2 \tag{2.2}$$

Equation (2.1) then takes the form

$$\frac{dX}{dY} = -Y \pm\left\{2\left[\delta\left(1+\frac{1}{M_0}\frac{\partial M_0}{\partial\tau}\right)+\frac{\partial\delta}{\partial\tau}\right]-(\mu+\mu_1)\right\}^{1/2} \tag{2.3}$$

On the shock wave front Huygens condition for the normal component of the velocity is automatically satisfied, inasmuch as in the entire flow M and p are connected by the relation $M = P/(nP_0)$.

The conditions for conservation across the front of the velocity component tangent to the shock front is of the type

$$a\psi - v = u_1(\psi + \vartheta + \alpha), \qquad u_1 = q_{1n}\cos\psi \approx q_{1n} \tag{2.4}$$

where α is the angle formed by the direction of the velocity field ahead of the wave front and the axis $\vartheta = 0$.

3. Let us examine the reflection of a plane, infinitely long wave OK having overpressure P_1 by a rigid wall with a small discontinuity in direction α (Fig.1). The magnitude of the break α coincides with the angle of incidence formed by the shock wave front and the normal to the wall at the point of intersection. Let this front propagate into undisturbed gas having zero overpressure. For a regular reflection (α greater than some critical angle α_*), the front of the reflected wave OE will consist, in the general case, of a straight line segment

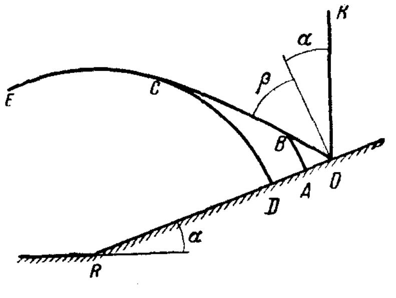

Fig. 1

OB with some constant pressure p_0 (the undisturbed front), a short arc BC where a rapid pressure drop from p_0 to P_1 occurs, and a circular arc CE, which is the front of the acoustic wave and along which the pressure hardly differs at all from that behind the incident front.

Thus, if the point R is chosen as the origin of coordinates and the axis $\vartheta = 0$ is directed along the wall, then in the region $ABCD$ we shall have a rapid variation of pressure both in the direction of the radius vector and perpendicular to it, i.e. a flow of the short wave type.

We now write out the boundary conditions of the problem. For the shock wave, using (2.4) and the notation of (1.14), we have

$$\mu_1 = \frac{M_1}{M_0}, \qquad \frac{\mu_1\psi}{\sqrt{{}^1/_2\,(n+1)\,M_0}} + \nu_1 = 0 \quad \text{on } OK \tag{3.1}$$

$$\mu = \frac{M}{M_0} \qquad \frac{(\mu-\mu_1)\,\psi}{\sqrt{{}^1/_2\,(n+1)\,M_0}} - \nu = \mu_1\left(\frac{\alpha}{\sqrt{{}^1/_2\,(n+1)\,M_0}} + Y\right) \quad \text{on } OE \tag{3.2}$$

Here, according to (2.1)

$$\psi = \left(\frac{n+1}{2}\,M_0\right)^{1/2}\left\{2\left[\delta\left(1+\frac{1}{M_0}\frac{\partial M_0}{\partial\tau}\right)+\frac{\partial\delta}{\partial\tau}\right]-(\mu+\mu_1)\right\}^{1/2} \tag{3.3}$$

At the front of the acoustic wave AB the velocity q may be considered with great accuracy as directed parallel to the wall, i.e.

$$\mu Y + \nu = 0, \quad \nu = 0 \quad \text{on the wall } DA \tag{3.4}$$

Finally, we require that in approaching the point C along BC the front of the reflected wave pass into the acoustic circle CE , i.e.

$$\mu = \mu_1 \quad \text{for} \quad \delta_1\left(1+\frac{1}{M_0}\frac{\partial M_0}{\partial\tau}\right)+\frac{\partial\delta_1}{\partial\tau} = \mu_1 \tag{3.5}$$

4. For the flow near the point O , Equations (3.1), (3.2) and (3.3) yield

$$\mu_1 = \frac{M_1}{M_0}, \qquad \alpha = \left(\frac{n+1}{2}M_0\right)^{1/2}\left\{2\left[\delta_0\left(1+\frac{1}{M_0}\frac{\partial M_0}{\partial\tau}\right)+\frac{\partial\delta_0}{\partial\tau}\right]-\mu_1\right\}^{1/2} \tag{4.1}$$

$$(1-\mu_1)\,\beta = \mu_1\alpha, \qquad \beta = \left(\frac{n+1}{2}\,M_0\right)^{1/2}\left\{2\left[\delta_0\left(1+\frac{1}{M_0}\frac{\partial M_0}{\partial\tau}\right)+\frac{\partial\delta_0}{\partial\tau}\right]-(1+\mu_1)\right\}^{1/2} \tag{4.2}$$

From these we obtain the known conditions [1] for β and M_0

$$\beta = \left(\frac{n+1}{2}\right)M_0\,\frac{\mu_1}{\sqrt{1-2\mu_1}}, \qquad M_0 = \frac{1-2\mu_1}{{}^1/_2\,(n+1)}\left(\frac{\alpha}{1-\mu_1}\right)^2 \tag{4.3}$$

The second equation of (4.1) gives the motion of the point O

$$\left(M_0+\frac{\partial M_0}{\partial\tau}\right)\delta_0 + M_0\frac{\partial\delta_0}{\partial\tau} = M_0\left(\frac{\mu_1}{2}+\frac{\alpha^{\circ 2}}{2}\right), \qquad \alpha^\circ = \frac{\alpha}{\sqrt{{}^1/_2\,(n+1)\,M_0}} \tag{4.4}$$

Substituting the value of μ_1 from (4.1) into the second equation of (4.3), we obtain the following relation for the determination of M_0:

$$M_0^2 - \left(\frac{2}{n+1}\alpha^2 + 2M_1\right)M_0 + \left(\frac{4}{n+1}\alpha^2 + M_1\right)M_1 = 0 \tag{4.5}$$

As is known form [1], of the two values of M_0, the actual flow corresponds to the one with the value

$$M_0 = \frac{\alpha^2 + (n+1) M_1 - \sqrt{\alpha^4 - 2(n+1)\alpha^2 M_1}}{n+1} \tag{4.6}$$

Then the maximum relative overpressure

$$\frac{M_0}{M_1} = \frac{1}{\mu_1} = \frac{1}{2}\alpha^{\vee 2} + 1 - \frac{1}{2}\alpha^{\vee}\sqrt{\alpha^{\vee 2} - 4}, \qquad \alpha^{\vee} = \frac{\alpha}{\sqrt{{}^1/_2 (n+1) M_1}} \tag{4.7}$$

assumes its largest value for $\alpha^{\vee} = 2$, and

$$\alpha \,/\, \sqrt{{}^1/_2 (n+1) M_1} = 2 \tag{4.8}$$

is the critical relation for regular reflection. Regular reflection must, therefore, be characterized by the condition $\alpha \geqslant 2\sqrt{{}^1/_2 (n+1) M_1}$, which is imposed on the original parameters M_1 and α, or equivalently, on p_1 and α. If for a definite M_1 the angle corresponding to the condition (4.8) is considered as the critical value of the angle and is denoted by α_*, then the condition of regular reflection is

$$\alpha \geqslant \alpha_* = 2\sqrt{{}^1/_2 (n+1) M_1} \tag{4.9}$$

We shall consider the intensity of the incident wave M_1 as constant. Then, according to (4.6), the value of M_0 is also constant and, therefore, all the parameters which characterize the pattern of the reflection are also constant. That is, in the variables μ, ν, δ, Y the pattern of the reflection is self-similar and Equation (1.15) can be used to describe the flow in the zone of the short wave. Equation (4.4) then takes the form

$$\delta_0 = {}^1/_2\mu_1 + {}^1/_2\alpha^{\circ 2} \tag{4.10}$$

The flow parameters β and M_0 are determined in accordance with (4.3), and from the first equation of (4.2) we have

$$\alpha^{\circ} = \frac{1 - \mu_1}{\sqrt{1 - 2\mu_1}} \tag{4.11}$$

We obtain

$$d\delta \,/\, dY = -\sqrt{2\delta - (\mu + \mu_1)} \tag{4.12}$$

for the front of the reflected wave.

This is used to determine the coordinates of the point B, the intersection of the reflected front with the acoustic circle. For the straight portion of the front, OB, taking into account that

$$\psi = \sqrt{{}^1/_2 (n+1) M_0}\, d\delta \,/\, dY = \beta - \vartheta, \qquad \alpha^2 - \beta^2 = {}^1/_2 (n+1) M_0$$

we obtain from (4.12)

$$\delta + \beta^{\circ} Y - {}^1/_2 Y^2 = {}^1/_2\mu_1 + {}^1/_2\alpha^{\circ 2}$$

The equation of the acoustic circle is $\delta = 1$. From this the coordinates of the point B are

$$Y_1 = \frac{\mu_1}{\sqrt{1 - 2\mu_1}} - \sqrt{1 - \mu_1}, \qquad \delta_1 = 1 \tag{4.13}$$

The region of constant pressure disappears if the front of the acoustic wave AB overtakes the front of the incident wave. This occurs when Y_1

becomes equal to zero. The coordinates of the point B will then be

$$Y_1 = 0, \qquad \delta_1 = \delta_0 = {}^1/_2\mu_1 + {}^1/_2\alpha^{\circ 2} \tag{4.14}$$

5. The system of equations of short waves (1.15) corresponds to Equation

$$\mu_\delta^2 + (\mu - \delta)\,\mu_{\delta\delta} + {}^1/_2\mu_{YY} + (k - 1)\,\mu_\delta = 0, \qquad k = {}^1/_2 \tag{5.1}$$

We seek a particular solution of this equation in the form $\mu = F(\zeta)$, $\zeta = \delta - cY^2$. This gives

$$\mu = a\delta - a\,(a - {}^1/_2)\,Y^2 + a_1 \tag{5.2}$$

where a and a_1 are arbitrary constants. According to the second equation of (1.17), we have

$$\nu = -\,2a\,(a - {}^1/_2)\,Y\delta + f\,(Y)$$

After substitution into the first equation this yields

$$f'\,(Y) = a\,(2a + 1)\,(a - {}^1/_2)\,Y^2 - (2a + 1)\,a_1$$

Then taking Equation (3.5) into account, we obtain

$$f\,(Y) = {}^1/_3 a\,(2a + 1)\,(a - {}^1/_2)\,Y^3 - (2a + 1)\,a_1 Y \tag{5.3}$$

$$\nu = {}^1/_3 a\,(2a + 1)\,(a - {}^1/_2)\,Y^3 - [2a\,(a - {}^1/_2)\,\delta + (2a + 1)\,a_1]\,Y$$

We shall now find the values of the constants a and a_1 in the solutions (5.2), (5.3). According to (4.12) and (4.13) we have at point B

$$a_1 = 1 - a\delta_1 + a\,(a - {}^1/_2)\,Y_1^2 \tag{5.4}$$

At point $C(Y_2, \delta_2)$, Equation (3.6) now provides

$$Y_2 = \left(\frac{a\,(\mu_1 - \delta_1) + 1 - \mu_1}{a\,(a - {}^1/_2)} + Y_1^2\right)^{1/2} \tag{5.5}$$

On AB the condition (3.4)

$$\begin{aligned} {}^1/_3 a\,(2a + 1)\,(a - {}^1/_2)\,Y^3 - [2a\,(a - {}^1/_2)\,\delta_1 + (2a + 1)\,(1 - a\delta_1)]\,Y - \\ - a\,(2a + 1)\,(a - {}^1/_2)\,Y_1^2 Y + Y = 0 \end{aligned} \tag{5.6}$$

is satisfied for $Y \leqslant Y_1$ up to terms of third order in the small Y_1. From Equation (3.2) at point C we find the value of the coefficient a

$$\begin{aligned} -\,{}^1/_3 a\,(2a + 1)\,(a - {}^1/_2)\,Y_2^3 + \{2a\,(a - {}^1/_2)\,\mu_1 + \\ + (2a + 1)\,[1 - a\delta_1 + a\,(a - {}^1/_2)\,Y_1^2]\}\,Y_2 = \mu_1\,(\alpha^\circ + Y_2) \end{aligned} \tag{5.7}$$

i.e. we have the final form of the solution

$$\begin{aligned} \mu &= a\,(\delta - \delta_1) - a\,(a - {}^1/_2)\,(Y^2 - Y_1^2) + 1 \\ \nu &= {}^1/_3 a\,(2a + 1)\,(a - {}^1/_2)\,Y^3 - \{2a\,(a - {}^1/_2)\,\delta + (2a + 1)\,[1 - a\delta_1 + \\ &\quad + a\,(a - {}^1/_2)\,Y_1^2]\}\,Y \end{aligned} \tag{5.8}$$

6. The equation of the reflected wave front

$$dX/dY = -Y - \sqrt{2\delta - (\mu + \mu_1)}$$

may now be integrated. Substituting μ from (5.8), we obtain

$$X'^2 + 2YX' + a(1-a)Y^2 - (2-a)X + \mu_1 + 1 - \\ - a\delta_1 + a(a - {}^1/_2)Y_1^2 = 0$$

The system of substitutions

$$X = \frac{x - \mu_1 + 1 - a\delta_1 + a(a - {}^1/_2)Y_1^2}{2-a}$$

$$x = Y^2 U(Y), \quad V^2 = 1 - a(1-a) + (2-a)U$$

reduces this equation to the form

$$\frac{2VdV}{2V^2 + (2-a)V - a(2a-1)} = -\frac{dY}{Y}$$

Integration yields

$$[(V-\alpha)Y]^\alpha [(V+\beta)Y]^\beta = A_0, \quad \begin{array}{l} \alpha = {}^1/_4 [\sqrt{17a^2 - 12a + 4} + (2-a)] \\ \beta = {}^1/_4 [\sqrt{17a^2 - 12a + 4} - (2-a)] \end{array}$$

or

$$Y = \frac{Z - z}{\alpha + \beta}, \quad V = \frac{Z\alpha + z\beta}{Z - z}, \quad Z = Az^{-\alpha/\beta} \tag{6.1}$$

From the condition that the front passes through the point O, we obtain for A

$$A = \{(2-a)\delta_0 - [\mu_1 + 1 - a\delta_1 + a(a - {}^1/_2)Y_1^2]\}^\lambda$$
$$\lambda = {}^1/_2 (\alpha + \beta)/\beta \tag{6.2}$$

The equation of the reflected wave in parametric form

$$Y = \frac{A - z^{2\lambda}}{(\alpha+\beta)z^{\alpha/\beta}}, \quad X = \frac{1}{2-a}\Big\{\mu_1 + 1 - a\delta_1 + a \\ (a - {}^1/_2)Y_1^2 + \left[\frac{\alpha A + \beta z^{2\lambda}}{(\alpha+\beta)z^{\alpha/\beta}}\right]^2 - [1 - a(1-a)]Y^2\Big\} \tag{6.3}$$

allows us to construct its front in a Cartesian coordinate system.

7. As an example we present the analysis of the patterns of reflection corresponding to the cases $\mu_1 = 0.4\ (\alpha > \alpha_*)$ and $\mu_1 = {}^1/_3\ (\alpha = \alpha_*)$.

For $\mu_1 = 0.4$, which gives $\alpha^\circ = 1.34, \alpha = 1.5\beta, (M_0 = 2.5\ M_1)$; he coordinates of O and B are: $O(\delta_0 = 1.1, Y_0 = 0)$ and $B\ (\delta_1 = 1,\ Y_1 = 0.12)$, respectively. According to (5.8) we have $a = 0.633$ and for point C $(\delta_2 = 0.4,\ Y_2 = 1.62)$.

For $\mu_1 = {}^1/_3$, we have $\alpha^\circ = 1.115,\ \alpha = 2\beta$, $(M_0 = 3M_1)$; the coordinates of point B are $B(\delta_1 = 0.83,\ Y_1 = 0)$. The value of $a = 0.792$ and for point C $(\delta_2 = {}^1/_3,\ Y_2 = 1.08)$.

In Figs 2 and 3 the calculated velocity fields are shown for these cases. That is, curves of equal μ , which correspond to curves of equal pressure, are shown and the curves of the reflected wave fronts are constructed.

The condition (3.2) of conservation of the tangential component of the velocity vector on crossing the reflected wave front can be considered to be satisfied accurately, for the error in fulfilling this condition nowhere exceeds 1% relative to the quantity $\mu_1(\alpha^\circ + Y)$

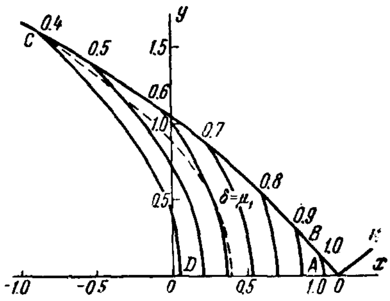

Fig. 2

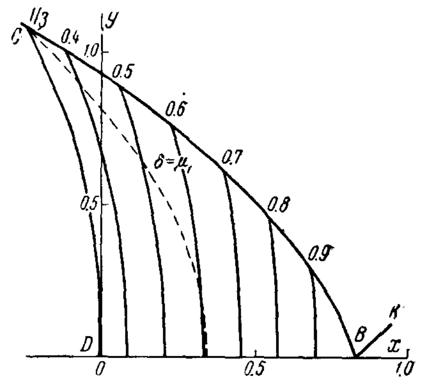

Fig. 3

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